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An Alternative Method to Estimate The Unknown Local Parameter α in Laplace Distribution by Using M.L.E

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Abstract
W. J. Hurley (2009) using a simple induction argument that depends only on knowing the shape of a function of sums of absolute values to calculate M.L.E , An Alternative method is presented here.

Keywords
MLE, median, double exponential (also referred to as Laplace distribution), the log of the random sample function that weight by The probability distribution function,likelihood function .

I. Introduction

W. J. Hurley (2009) in article (An Inductive Approach to calculate M.L.E) Depends upon the function shape to determine the M.L.E, principle agree with Hurley's but it used special technique that shows more results that it couldn't be show in Hurley's; this is emphasized here

II. Methodology

The double exponential distributionis :

$f(x) = \frac{1}{2} e^{-|x_i - \alpha|}$ $-\infty < x_i < \infty$

likelihood function of double exponential distribution (Can we say the likelihood function is the log of the random sample function that weight by The double exponential function or In general it is the log of the random sample function that weight by The probability distribution function!?) is given by:

$$lnf(x_1, x_2, x_3, \dots, x_n;\ \alpha) = -nln2 - \sum_{i=1}^{n} |x_i - \alpha|$$

$$lnf(x_1, x_2, x_3, \dots, x_n;\ \alpha) + nln2 = -\sum_{i=1}^{n} |x_i - \alpha|$$

$$\varphi(x_1, x_2, x_3, \dots, x_n;\ \alpha) = -\sum_{i=1}^{n} |x_i - \alpha|$$

The function can be written as follows:

$$\varphi(x_1, x_2, x_3 \dots, x_n;\ \alpha) = -[\sum_{L=1}^{k} (y_L + \alpha) + \sum_{j=1}^{m} (z_j - \alpha)]$$

Where:

$$|x_i - \alpha| = \begin{cases} (z_j - \alpha) & \text{if } x_i > \alpha \quad \text{i.e.: } z_j = x_i \\ (y_L + \alpha) & \text{if } x_i < \alpha^* \quad \text{i.e.: } y_L = -x_i \end{cases}$$

*Since parameter is not equal estimator then the equality has been deleted .

& we can write the domain of this function as:

$-\infty < g_1 < g_2 \dots < g_m < \alpha < d_1 < d_2 \dots < d_k < \infty$; $k > m$ or $k = m$ or $k < m$.

and

g_1= *smallest of* $(-y_1, -y_2, -y_3, \dots, -y_L, \dots, -y_k)$.

g_2= *second smallest of* $(-y_1, -y_2, -y_3, \dots, -y_L, \dots, -y_k)$.

.

g_q=*q^th smallest of* $(-y_1, -y_2, -y_3, \dots, -y_L, \dots, -y_k)$.

g_k= *largest of* $(-y_1, -y_2, -y_3, \dots, -y_L, \dots, -y_k)$.

d_1= *smallest of* $(z_1, z_2, z_3, \dots, z_j, \dots, z_m)$.

d_2= *second smallest of* $(z_1, z_2, z_3, \dots, z_j, \dots, z_m)$.

.

d_p=*p^th smallest of* $(z_1, z_2, z_3, \dots, z_j, \dots, z_m)$.

d_m= *largest of* $(z_1, z_2, z_3, \dots, z_j, \dots, z_m)$.

m is #($\alpha > x_i$) ; k is #($\alpha < x_i$) ; $m + k = n$.

$if\ \ k > m$

That is:

$$\varphi(x_1, x_2, x_3, \dots, x_n;\ \alpha) = -[\sum_{L=1}^{k} (y_L + \alpha) + \sum_{j=1}^{m} (z_j - \alpha)]$$

we can write the domain of this function as:

$-\infty < g_1 < g_2 \dots < g_m < \alpha < d_1 < d_2 \dots < d_k < \infty$; $k > m$.

we note:

$$w(m = 0, k) < w(m = 1, k) < w(m = 2, k) < \dots < w(m = n - k, k)\ ;\ k > m .$$

where: $w(m, k; \alpha)$ present $\varphi(x_1, x_2, x_3, \dots, x_n;\ \alpha)$; $k > m$.

& the estimator is:

$if w(m = 0, k)$, the estimator is $\hat{\alpha} = d_1$.

$if w(m = 1, k)$, the estimator is $\hat{\alpha} = g_1$.

$if w(m = 2, k)$, the estimator is $\hat{\alpha} = g_2$.

.

$if w(m = n - k, k)$, the estimator is $\hat{\alpha} = g_{n-k}$; $k > m$.

$if\ \ k = m$

That is:

$$\varphi(x_1, x_2, x_3, \dots, x_n;\ \alpha) = -[\sum_{L=1}^{k} (y_L + \alpha) + \sum_{j=1}^{m} (z_j - \alpha)]$$

& we can write the domain of this function as:

$-\infty < g_1 < g_2 \dots < g_m < \alpha < d_1 < d_2 \dots < d_k < \infty$; $k = m$.

the estimator is $\hat{\alpha} = h\, g_m + (1 - h) d_1$ for α ; $0 \le h \le 1$; $k = m$.

$if k < m$

That is:

$$\varphi(x_1, x_2, x_3, \dots, x_n;\ \alpha) = -[\sum_{L=1}^{k} (y_L + \alpha) + \sum_{j=1}^{m} (z_j - \alpha)]\ ;$$

& we can write the domain of this function as:

$-\infty < g_1 < g_2 \dots < g_m < \alpha < d_1 < d_2 \dots < d_k < \infty$; $k < m$.

we note:

$$w(m, k = 0) > w(m, k = 1) > w(m, k = 2) > \dots > w(m, k = n - m);\ k < m .$$

where: $w(m, k; \alpha)$ present $\varphi(x_1, x_2, x_3, \dots, x_n;\ \alpha)$; $k < m$.

& the estimator is:

$if w(m, k = 0)$, the estimator is $\hat{\alpha} = g_m$.

$if w(m, k = 1)$, the estimator is $\hat{\alpha} = d_1$

$if\ w(m, k = 2)$, the estimator is $\hat{\alpha} = d_2$

.

$if w(m, k = n - m)$, the estimator is $\hat{\alpha} = d_{n-m}$; $k < m$.

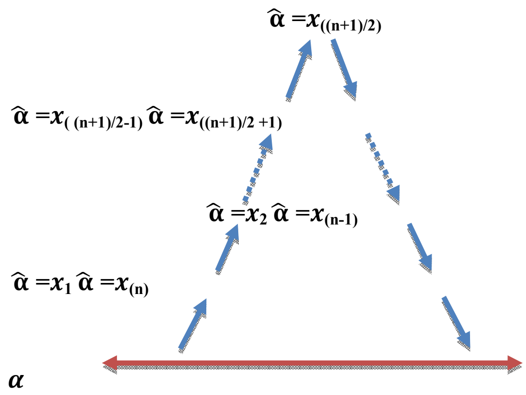
III. Conclusions

The maximum likelihood estimator for Laplace distribution to calculate $\hat{\alpha}$ be as follow :

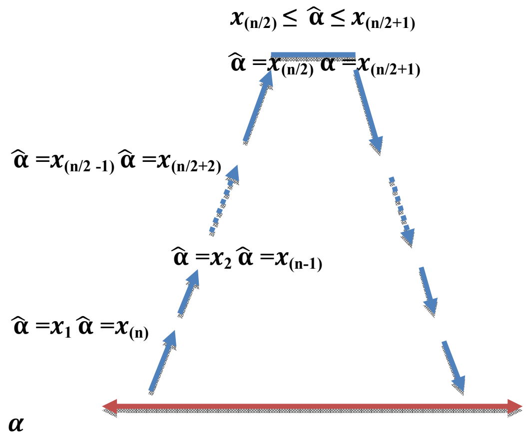
$$\therefore \hat{\alpha} = \begin{cases} h\, x_{(n/2)} + (1-h) x_{(n/2+1)} \;;\; 0 \le h \le 1 & iff\, n \text{ is even} \\ x_{((n+1)/2)} & iff\, n \text{ is odd} \end{cases}$$

Because the estimator make the function greatest possible & $\{x_1, x_2, \dots, x_n / m.l.e\ above\}$ also estimators but not make the function greatest possible& $\hat{\alpha} = x$ when $n = 1$.

And the chart as **:** Suppose that $x_1 < x_2 < x_3 < \dots < x_{(n-2)} < x_{(n-1)} < x_{(n)}$; $x_i < \alpha$ or $x_i > \alpha$; $-\infty < \alpha < \infty$



n is odd number



n is evennumber

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