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A Novel Approach for Solving Fuzzy Linear programming Problem using Pentagonal Fuzzy numbers

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Abstract

A new approach for solving fully fuzzy linear programming problems (FFLPP) is proposed, based upon the new Ranking function. Here, the given FFLP is converted into Fuzzy Variable Linear Programming Problem (FVLPP) using Pentagonal Ranking technique and the optimal solution is obtained and ranking is compared. Numerical examples are used to demonstrate the effectiveness and accuracy of this method.

Keywords

Pentagonal fuzzy number, Ranking function, Fuzzy Variable Linear programming,(FVLPP)

I. Introduction

Ranking fuzzy number is used mainly in decision-making, data analysis, artificial intelligence and various other fields of operation research. In fuzzy environment ranking fuzzy numbers is a very important decision making procedure. The concept of Fuzzy Linear Programming (FLP) was first introduced by Tanaka et al. [8, 9]. Chanas [2] proposed a fuzzy programming in multiobjective linear programming. Allahviranloo et al. [1] proposed a new method for solving fully fuzzy linear programming problems by the use of ranking function. Helen & .Uma [5] proposed new ranking and operation on pentagonal fuzzy number. Dhurai and Karpagam [4] introduced new method to solve FFLPP using three ranking techniques. Nasseri et.al [7] proposed a new method for solving fuzzy linear programming problems in which he has used the fuzzy ranking method for converting the fuzzy objective function into crisp objective function. Chandrasekaran [3] used a centroid based distance method to rank pentagonal fuzzy numbers. Kolman and Hill was introduced a FFLP problem. .Karpagam and Sumathi [6] introduced FVLP problem. This paper describes a new method to solve FFLPP and comparative study is made between three different rankings.

II. Preliminaries

1. Definition [5]

A fuzzy set is characterized by a membership function mapping the elements of domain, space or universe of discourse x to the unit interval [0, 1]. A fuzzy set $\tilde{A}$ is set of ordered pairs $\{x, \mu_{\tilde{A}}(x) / x \in R\}$ Where $\mu_{\tilde{A}}(x) : R \rightarrow [0,1]$ is upper semi- continuous function $\mu_{\tilde{A}}(x)$ is called membership function of the fuzzy set.

2. Definition [5]

A fuzzy number $\tilde{A}$ in the real line R is a fuzzy set

$\mu_{\tilde{A}}(x) : R \rightarrow [0,1]$ that satisfies the following properties.

(i) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

(ii) There exists an $x \in R$ such that $\mu_{\tilde{A}}(x)$ =1.

(iii) $\mu_{\tilde{A}}(x)$ is convex if $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min \{\mu_{\tilde{A}}(x_1)\ \mu_{\tilde{A}}(x_2)\}\ \forall x_1, x_2 \in X, \lambda \in [0,1]$

3. Definition [5]

A pentagon fuzzy number $\tilde{A}_P$ = (a_1 ,a_2 ,a_3 ,a_4 ,a_5) where a_1 ,a_2 ,a_3 ,a_4 ,a_5 are real numbers and its membership function is given below.

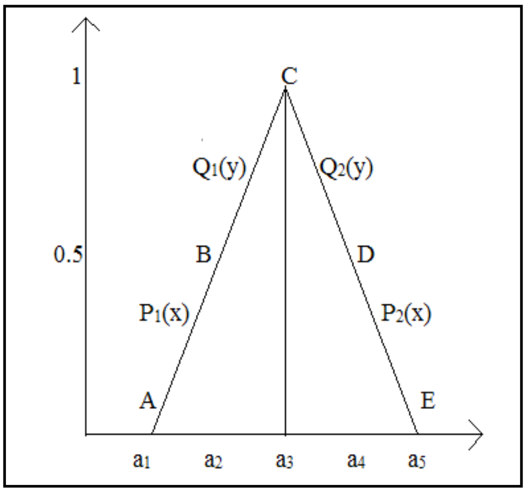

Fig: 2.1 : Graphical representation of a normal pentagon fuzzy number for x $\in$[0, 1]

4. Definition [4]

A linear programming problem is called fuzzy variable linear programming problem (FVLPP), if some of the parameters are crisp, and variables and right hand sides are fuzzy numbers. General form of FVLPP as follows:

Maximize (or Minimize) $\tilde{z} = \tilde{c}\ \tilde{x}$

Subject to,

$\tilde{A} \otimes \tilde{x}\ \{\leq, \approx, \geq\}\ \tilde{b}$, -------------- (2.1)

$\tilde{x} \geq 0$ are integers. -------------- (2.2)

Where $\tilde{c} \in R^n$, $\tilde{A} \in (R)^{m\times n}$, $R_i \in (F(R))^m$ and $\tilde{x} \in (F(R))^n$.

5. Definition [4]

An effective approach for ordering the elements of F(R) is also to define a ranking function

$R_i(\tilde{A}_P)$:F(R) $\rightarrow$ R which maps each fuzzy number into the real line, where a natural order exists.

We define orders on F(R) by:

$\tilde{A}_P \geq \tilde{B}_P$ if and only if $R_i(\tilde{A}_P) \geq R(\tilde{B}_P)$

$\tilde{A}_P \leq \tilde{B}_P$ if and only if $R_i(\tilde{A}_P) \leq R(\tilde{B}_P)$,

$\tilde{A}_P = \tilde{B}_P$ if and only if $R_i(\tilde{A}_P) = R(\tilde{B}_P)$.

III. Proposed Ranking functions

In this paper, the pentagon has been divided into three plane figures .Then the robust ranking function is taken for these three plane figures. Let $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ be pentagon fuzzy numbers. The ranking functions are obtained as below:

1. Ranking method (i):

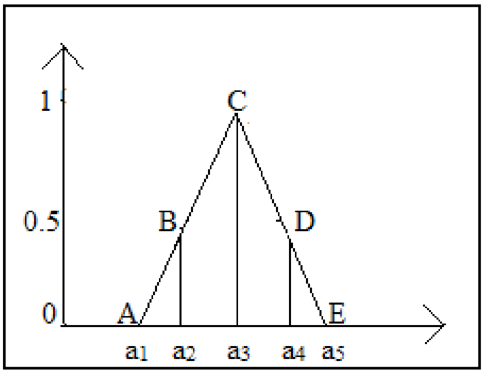

Fig 3.1

Pentagon is divided into four parts which includes two triangles and two trapezoids. By using robust ranking technique for triangle and trapezoidal fuzzy numbers here new ranking was introduced.

$$R_1(\tilde{A}_P) = \int_0^1 \{(a_2 - a_1)\alpha + a_1, a_5 - (a_5 - a_4)\alpha\} 0.5\, d\alpha \cdot$$

2. Ranking method (ii):

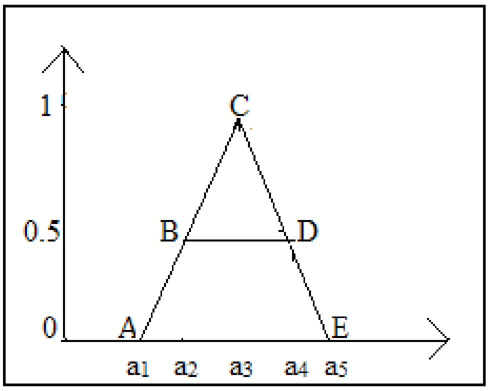

Fig 3.2

Pentagon is divided into two parts which includes a triangle and a trapezoid. By using robust ranking for triangle and trapezoidal fuzzy numbers here new ranking was produced.

$$R_2(\tilde{A}_P) = \int_0^1 \begin{Bmatrix} (a_2 - a_1)\alpha + a_1 + (a_3 - a_2)\alpha + a_2, \\ a_4 - (a_4 - a_3)\alpha + a_5 - (a_5 - a_4)\alpha \end{Bmatrix} 0.5\, d\alpha$$

3. Ranking method (iii):

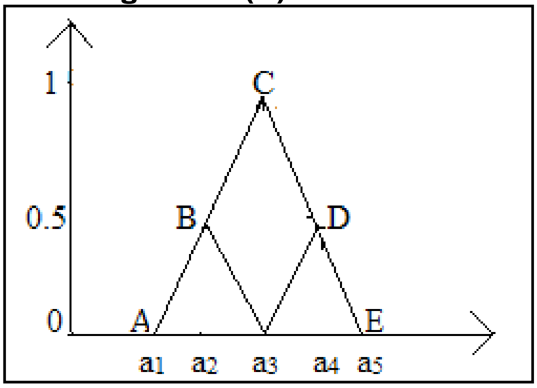

Fig 3.3

Pentagon is divided into three parts which includes two triangles and a trapezoidal. By using robust ranking for triangle and trapezoidal fuzzy numbers here new ranking was produced.

$$R_3(\tilde{A}_P) = \int_0^1 \begin{Bmatrix} (a_4 - a_1)\alpha + a_1 + a_2 + a_3, \\ a_3 + a_4 + a_5 - (a_5 - a_2)\alpha \end{Bmatrix} 0.5\, d\alpha \, .$$

IV. Fully Fuzzy linear programming problems (FFLPP)[

Consider the following fully fuzzy linear programming problems:

Maximize (or Minimize) $\tilde{z} = \tilde{c}^T \tilde{x}$

Subject to,

$\tilde{A} \otimes \tilde{x} \; \{\leq, \approx, \geq\} \; \tilde{b}$, --------------(2.1) $\tilde{x} \geq 0$ and are integers. ------------- (2.2)

Where the cost vector $\tilde{c}^T = (\tilde{c}_j)_{1\times n}$, $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$, $\tilde{x} = (\tilde{x}_j)_{n\times 1}$ and $\tilde{b} = (\tilde{b}_i)_{m\times 1}$ and $\tilde{a}_{ij}$, $\tilde{x}_j, \tilde{b}_i$, $\tilde{c}_j \in F(R)$ for all $1 \leq j \leq n$ and for all $1 \leq i \leq m$.

1. Proposed Algorithm

Step1:

Formulate the chosen problem in to the following fuzzy LPP as

$$\text{Maximize } \tilde{z} = \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j$$

Subject to,

$$\sum_{j=1}^{n} a_{ij} \tilde{x}_j \leq, \cong, \geq \tilde{b}_i \quad i = 1,2,3 \ldots m \,,$$

$$\tilde{x}_j \geq 0 \quad , \quad j = 1,2,3 \ldots n$$

Step 2:

Using the Ranking functions, the FFLPP transformed into FVLPP.

Step 3:

The FVLPP is solved by using simplex method to obtain the optimal solution.

V. Numerical Example

Consider the following fully fuzzy linear programming problem

Maximize $\widetilde{z} \approx (0.8, 0.7, 0.5, 0.3, 0.2) \otimes \widetilde{x}_1 + (0.2, 0.3, 0.4, 0.1, 0.2) \otimes \widetilde{x}_2$

subject to,

$(0.2,0.4,0.5,0.6,0.7) \otimes \widetilde{x}_1 + (0.3,0.2,0.6,0.5,0.1) \otimes \widetilde{x}_2 \leq (0.1,0.2,0.5,0.4,0.3)$;

$(0.7,0.8,0.6,0.9,0.1) \otimes \widetilde{x}_1 + (0.2,0.3,0.5,0.7,0.1) \otimes \widetilde{x}_2 \leq (0.2,0.3,0.5,0.7,0.9)$;

$$\widetilde{x}_1, \widetilde{x}_2 \geq 0.$$

Solution:

Step 1:

Formulate the numerical problem using the three ranking methods:

Ranking method (i):

Maximize $\widetilde{z} \approx 0.5 \otimes \widetilde{x}_1 + 0.2 \otimes \widetilde{x}_2$

subject to,

$$0.475 \otimes \widetilde{x}_1 + 0.275 \otimes \widetilde{x}_2 \leq 0.3;$$

$$0.625 \otimes \widetilde{x}_1 + 0.325 \otimes \widetilde{x}_2 \leq 0.5;$$

$$\widetilde{x}_1, \widetilde{x}_2 \geq 0.$$

Ranking method (ii):

Maximize $\widetilde{z} \approx 1 \otimes \widetilde{x}_1 + 0.4 \otimes \widetilde{x}_2$

subject to,

$$0.925 \otimes \widetilde{x}_1 + 0.3 \otimes \widetilde{x}_2 \leq 0.6;$$

$$1.3 \otimes \widetilde{x}_1 + 0.825 \otimes \widetilde{x}_2 \leq 1.0475;$$

$$\widetilde{x}_1, \widetilde{x}_2 \geq 0.$$

Ranking method (iii):

Maximize $\widetilde{z} \approx 1.5 \otimes \widetilde{x}_1 + 0.8 \otimes \widetilde{x}_2$

subject to,

$$0.475 \otimes \widetilde{x}_1 + 1.225 \otimes \widetilde{x}_2 \leq 1.1;$$

$$2.025 \otimes \widetilde{x}_1 + 1.325 \otimes \widetilde{x}_2 \leq 1.525;$$

$$\widetilde{x}_1, \widetilde{x}_2 \geq 0.$$

Step 2:

Ranking method(i) we obtain

$\widetilde{x}_1 \approx 0.526$, $\widetilde{x}_2 \approx 0$

Maximize $\widetilde{z} \approx 0.2428$

Ranking method(ii) we obtain

$\widetilde{x}_1 \approx 0.667$, $\widetilde{x}_2 \approx 0$

Maximize $\widetilde{z} \approx 0.667$

Ranking method(iii) we obtain

$\widetilde{x}_1 \approx 0.7478$, $\widetilde{x}_2 \approx 0$

Maximize $\widetilde{z} \approx 1.155$

VI. Comparison study:

Table 6.1 : Comparison Table

RANKING METHOD	X_1	X_2	Z
Method(i)	0.526	0	0.2428
Method(ii)	0.667	0	0.667
Method(iii)	0.7478	0	1.155

1. Comparison Chart

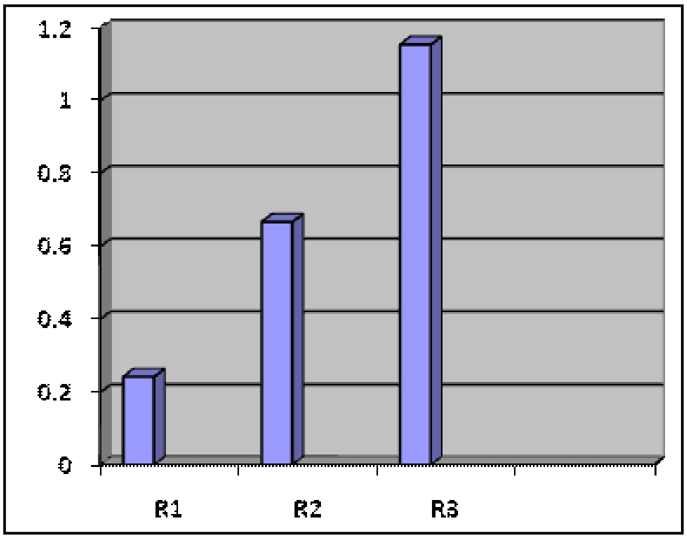

Fig. 6.1 : Comparison Chart

VII. Conclusion

In this paper a new method is proposed for solving where the FFLP problem is converted into FVLP problem using new Ranking function. We have obtained the maximum results by using the third ranking method (R3) compared to the other two ranking methods (R1) and (R.2). Ranking function is reasonable and effective for calculating the pentagonal weights of criteria. Therefore it is easier to solve fully fuzzy linear programming problem.

References

[1] *T.Allahviranloo, F.H.Lotfi, M.K.Kiasary, N.A.Kiani and L. Alizadeh, "Solving fully fuzzy linear Programming problems by the ranking function", Applied Mathematical Sciences,2 (2008), 19-32.*

[2] *D.Chanas, "Fuzzy programming in multiobjective linear programming a parametric approach", Fuzzy Sets and Systems, 29 (1989) 303-313.*

[3] *S.Chandrasekaran and A.Juno Saju "An approach to solve Multi Objective Fuzzy linear Programming", Engineering & Technology, Vol. 3, ISSN 2348 – 7968*

[4] *K.Dhurai and A.Karpagam, "Fuzzy optimal solution for fully fuzzy linear programming problems using hexagonal fuzzy numbers", International Journal of Fuzzy Mathematical Archive, 2(2016) 117-123.*

[5] *R.Helen and G.Uma, "A new operation and ranking on pentagon fuzzy numbers", International Journal of*

Mathematical Sciences & Applications ,2 (2015) 341-346.

[6] *A.Karpagam and P.Sumathi, "Innovative Method For Solving Fuzzy Linear Programming Problems With Symmetric Trapezoidal Fuzzy Numbers", International Journal Of Latest Research Science And Technology, 3(2014) 95-98.*

[7] *S.H.Nasseri, "A new method for solving fuzzy linear programming by solving linear programming", Applied Mathematical Sciences, 2 (2008) 37-46.*

[8] *H.Tanaka and K.Asai, "Fuzzy linear programming problems with fuzzy numbers", Fuzzy Sets and Systems, 13 (1984) 1-10.*

[9] *H.Tanaka, T.Okuda and K.Asai, "Fuzzy mathematical programming", Journal of Cybernetics and Systems, 3 (1973) 37-46.*