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Stability Control of Inverted Pendulum Based on Fractional Order PID Controller

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Abstract

The inverted pendulum is a classical control problem, which involves developing a system to balance a pendulum. Generally, PID Controllers are widely used for control applications. The performance of PID Controller can be improved by appropriate setting of fractional integral and derivative actions. In this paper modeling of an inverted pendulum has been done and then different controllers have been used for stabilization of the pendulum. The design methods of integer order controllers and fractional order controllers are given. The majority of tasks is carried out by means of the FOMCON ("Fractional –Order Modeling and Control") toolbox running in the MATLAB computing environment. The simulation results prove that the proposed method can achieve high performance comparing the Integer order PID Controller, as whole the Fractional Order PID Controller is the best controller.

Keywords

Inverted Pendulum, fractional order PID Controllers, FOMCON toolbox.

I. Introduction

The inverted pendulum system features as higher order, non-linear, strong coupling and multivariate, which has been studied by many researchers. It is used to model the field of robotics and aerospace field, and so has important significance both in the field of the theoretical study research and practice.

The theory of fractional calculus refers to the fractional order differentiation and integration. The fractional controller, first proposed by I.Podlubny, is different from the integer order controller in the point that it adds two parameters: the fraction differential operator and integration operator, which leads to fractional order PID Controller more flexible to control the plant.

Proportional-Integral-Derivative (PID) Controllers have been used for several decades in industries for process control applications. The reason for their wide popularity lies in the simplicity of design and good performance including low percentage overshoot and small settling time. In FOPID Controller Integral and Derivative operations are usually of fractional order, therefore in addition of setting the proportional, derivative and integral constants K_p,K_D,K_I there are two more parameters: the order of fractional integration λ and fractional derivative μ.

In general, the integer-order approximation of the fractional systems can cause significant differences between mathematical model and real system. The main reason for using integer-order models was the absence of solution methods for fractional differential equations. At present time there are lots of methods for approximation of fractional derivative and integral and fractional calculus can be easily using wide areas of applications. So the problem is overcome by the use of fractional order controllers.

The one-dimensional swinging inverted pendulum with two degrees of freedom (i.e. the angle of the inverted pendulum and the movement of robot along forward and backward direction) is a popular demonstration of using feedback control to stabilize an open-loop unstable system.

In this system, an inverted pendulum is attached to a cart equipped with a motor that drives it along a horizontal track. The thin vertical rod (the pendulum) hinged at the bottom, referred to as pivot point is mounted on a cart which can move along horizontal direction. The cart, depending upon the direction of the deflection of the pendulum (the angle of the inverted pendulum), moves horizontally in order to bring the pendulum to absolute rest in a vertical position.

A FOPID (Fractional Order PID) controller has been used to generate signal to control the speed and direction of the motor. The sensor used in this work for generating the appropriate control signal.

The Mathematical expression was established to find the system transfer function based on Newton's second law of motion. MATLAB has been used for closed loop transfer function simulation with various controllers and comparing their results, finding the best controller.

II. Theory of Fractional Order

There are several definitions of fractional differentiation and integral such as Riemann-Liouville definition, Grunwald-Letnikov definition and Caputo definition, which are commonly used in theory and automatic control field.

The integral definition of Riemann-Liouville follows as:

$$_aD_t^{-n} f(t) = (1/(n-1)!)\int_0^{t-a} z^{n-1}\, f(t-z)dz$$

So the derivative definition of Grunwald- Letnikov follows as:

$$f(t) = (t-a)^v$$

$$_aD_t^p(t-a)^v = (1/\,\Gamma(-p))\int_a^t (t-\tau)^{-p-1}(\tau-a)^v\, d\tau$$

v- is a real number.

III. Inverted Pendulum System Modeling and Analysis

1. Inverted pendulum system

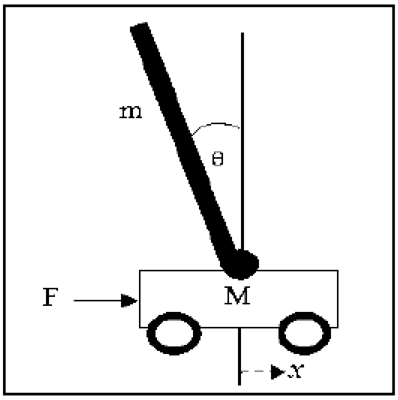

Fig.1: Cart and Inverted Pendulum System

To find the system transfer function a brief description on the modeling of the inverted pendulum is presented in this section. The system consists of an inverted pole hinged on a cart which is free to move in the x direction as shown in Fig.1.

In order to obtain the system dynamics the following assumptions have been made:

1. The system starts in an equilibrium state i.e. that the initial conditions are assumed to be zero.
2. The pendulum does not move more than a few degrees away from the vertical to satisfy a linear model.
3. A step input (displacement of the pendulum, θ) is applied to the system.

For the analysis of system dynamic equations, Newton's second law of motion was applied. Fig. 2 represents the Free Body Diagram (FBD) of the mechanism. The Force Distribution of the mechanism is shown in Fig. 2.

Table.1: Parameters of the Inverted Pendulum

M	Mass of the Cart	0.5kg
M	Mass of the pendulum	0.2kg
B	Friction of the cart	0.1
L	Length of the pendulum	0.3m
I	Inertia of the pendulum	$0.006\ kg.m^2$
F	Force applied to the cart	
G	Gravity	$9.8\ m/s^2$
□	Vertical pendulum angle	

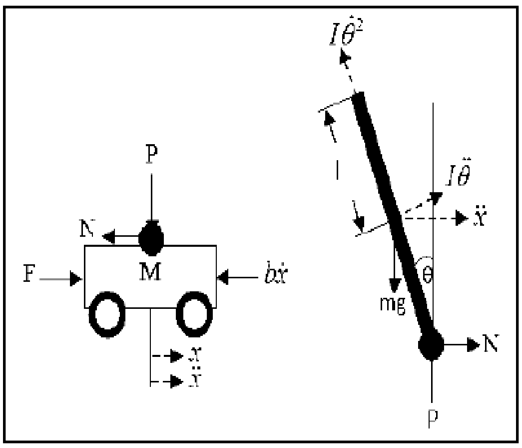

Fig.2: Free body diagram of the Inverted Pendulum

x, x', x'' - Cart position coordinate, cart velocity and cart acceleration, respectively.

$\theta, \theta', \theta''$ - Pendulum angle from the vertical, pendulum angular velocity and angular acceleration, respectively.

N – Sum of the forces of the cart.

P – Sum of the forces of the pendulum in the horizontal direction.

While the pendulum rod tilts with some angle, it resolves two force components along horizontal and vertical direction. 'P' denotes the force exerted by the pendulum in vertical direction, and 'N' in horizontal direction, when θ = 90 deg, N= 0, and P = maximum.

Summing the forces in the Free Body Diagram of the cart in the horizontal direction, you get the following equation of motion:

$Mx''+bx'+N = F$ (1)

The sum of forces in the vertical direction is not considered because there is no motion in this direction and we consider that the reaction force of the earth balances all the vertical forces.

Summing the forces along the horizontal direction,

$N = mx'' + mL\theta''\cos\theta - mL\theta'^2\sin\theta$ (2)

After substituting eqn. 2 into eqn. 1, the first equation of motion for the system was found as follows:

$F=(M+m)x''+bx'+mL\theta''\cos\theta-mL\theta'^2\sin\theta$(3)

To obtain the second equation of motion, the forces along the perpendicular direction of the pendulum was summed up.

$P\sin\theta+N\cos\theta–mg\sin\theta=mL\theta''+mx''\cos\theta$(4)

To get rid of P and N terms from the eqn. 4, the moments around the centroid of the pendulum was taken which resulted following equation:

$- PL\sin\theta– NL\cos\theta = I\theta''$ (5)

Combining eqn. 4 and 5, the second dynamic equation was obtained as follows:

$(I + mL^2)\theta'' + mgL\sin\theta = - mLx''\cos\theta$(6)

(3) & (6) are non-linear and need to be linearized for the operating range. Since the pendulum is being stabilized at an unstable equilibrium position, which is Л radians from the stable equilibrium position, this set of equations should be linearized about θ = Л. Assume that θ = Л + □, (where □ represents a small angle from the vertical upward direction). Therefore, cos (θ) = -1, sin (θ) = - □, and (d(θ)/dt)^2 = 0.

Thus, after linearization the following 2 equations of motion were appeared (where F represents the input):

$(M + m)x'' + bx' - mL□'' = F$

$(I + mL^2)□'' - mgL□= mLx''$

Taking Laplace transform,

$(M + m)X(s)s^2 + bX(s)s –mL□(s)s^2 = F(s)$

$(I + mL^2) □(s)s^2–mgL□(s) = mLX(s)s^2$

Assuming that the initial condition is 0, then the transfer function of the inverted pendulum is nothing but the ratio of Laplace Transform of angle of the pendulum [□(s)] and the external force [F(s)]. It is denoted as P(s).

$P(s) = □(s) / F(s) = ((mL/q)s) / (s^3 + (b (I + mL^2)/q) s^2 - ((M + m) mgL/q)s - (bmgL/q))$

$q = (M + m) (I+mL^2) – (mL)^2$

Applying the values for the parameters, the transfer function of the plant becomes,

$P(s) = (4.5454s) / (s^3 + 0.1818s^2- 31.1818s -4.4545)$

2. Analysis of uncompensated system

Fig.3 shows the unit step response of the system, which shows that amplitude of the system tends to infinity, so the system is

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unstable.

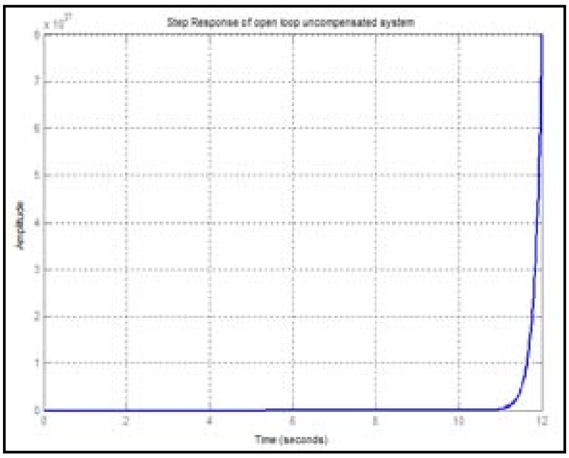

Fig.3: Open loop step response of uncompensated system

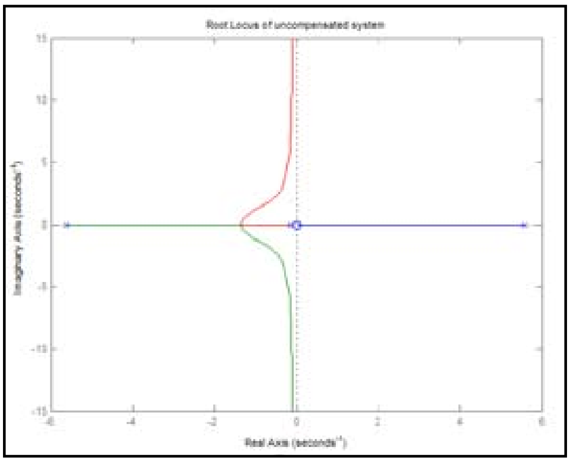

Fig.4: Root locus of uncompensated system

The poles position of the Inverted Pendulum (in open loop configuration) shows that the system is unstable, as one of the poles of the transfer function lies on the Right Half Side of the s-plane. Thus the system is absolutely unstable.

From Fig 3 & 4, it is revealed that the inverted pendulum system is unstable.

3. Stabilization of inverted pendulum system

P(s) = (4.5454s) / (s-5.5651) (s+5.6041) (s+0.1428)

Observation of the closed loop unity feedback response to check the stability:

Many systems are unstable in open loop but stable in closed loop configuration. The closed loop uncompensated system can be studied by viewing the root locus plot of the system. Following figure shows the root locus plot of the system.

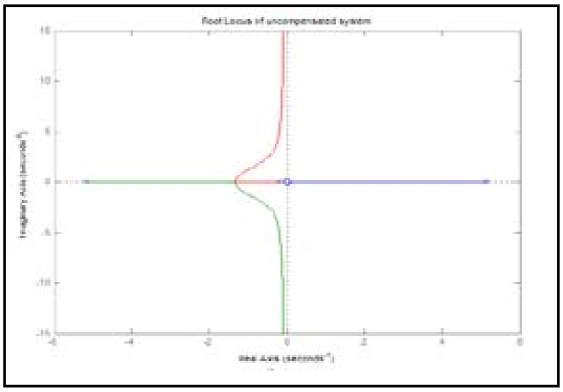

Fig.5: Root locus of closed loop uncompensated system

The plot reveals that the system cannot be controlled by a simple unity feedback loop. Whatever be the value of loop gain, one branch of the locus remains on RHS (in unstable region) of s-plane. This makes control impossible by unity feedback.

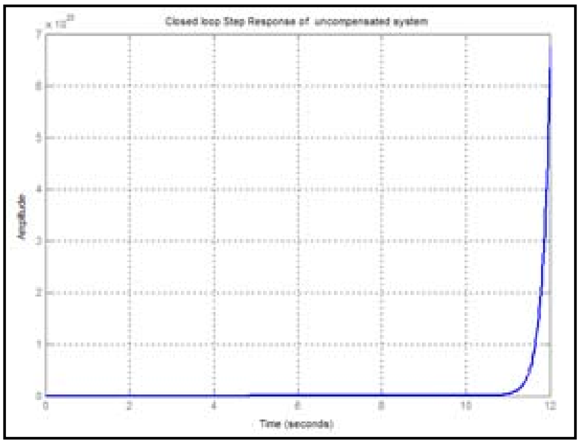

Fig.6: Closed loop step response of uncompensated system

From the above analysis, it is concluded that using only the gain compensation in closed loop cannot control the Inverted Pendulum. RESHAPING OF THE SYSTEM ROOT LOCUS is necessary so that the system has all its roots in the left half plane (stable region) of the s-plane.

4. Compensator Design

Direct compensator is used to eliminate the unstable zero-pole of the system. The compensation unit function is denoted as H(s).

H(s) = (s-5.5651)/s

Now the generalized plant is $P_1(s)$.

$P_1(s)$ = 4.5454 / (s+5.6041) (s+0.1428)

$P_1(s)$ = 4.5454 / (s^2 + 5.7469s + 0.80026)

The unit step response of the generalized plant is shows in Fig.8. It shows that generalized plant under the unit step signal, directly eliminate the open loop unstable zero pole. Although the system achieve stability, the system has lost accuracy index, rapidity and robustness performance indicators.

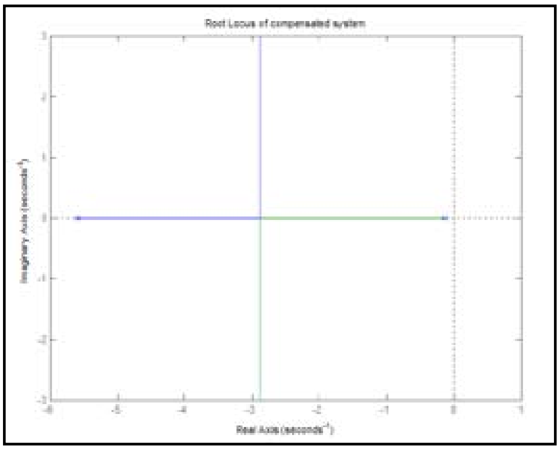

Fig.7: Root locus of compensated system

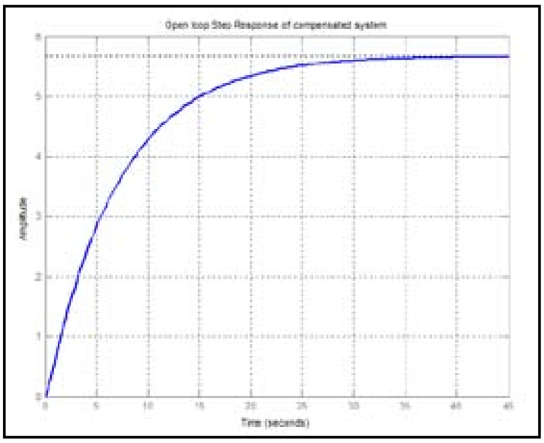

Fig.8: Open loop step response of compensated system

Even after applying unity feedback also, it does not provide the desired output. It is shown below.

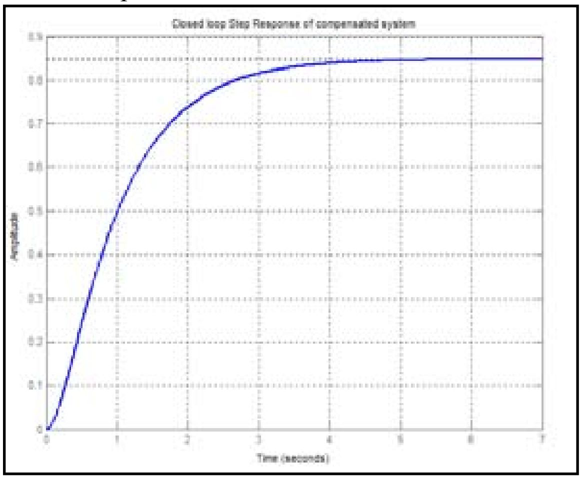

Fig.9: Closed loop step response of compensated system

Now the system is stable and can use the various control method to achieve better performance.The basic block diagram is shown below.

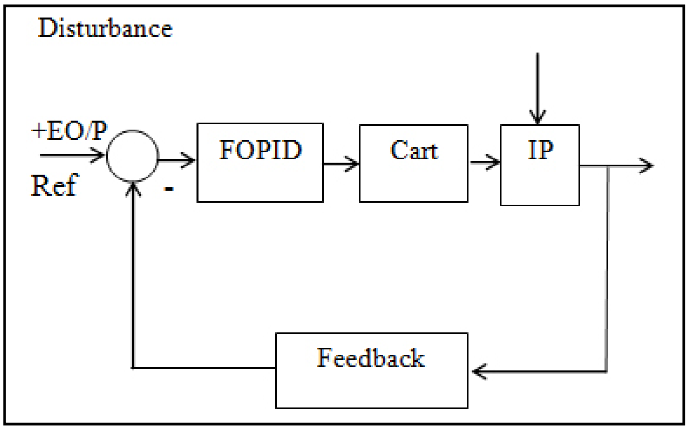

Fig.10 Block diagram of Inverted Pendulum System

Where,

- Ref – Reference
- E – Error
- IP – Inverted Pendulum
- O/P – Output
- FOPID – Fractional Order PID Controller

5. Various controllers

The controllers which are given in Table.2 are used for achieving better performance of the inverted pendulum and its design parameters are given which is found out by hit and trial method.

Table.2: Controller Design

CONTROLLER	DESIGN
Integer Order P controller (IOP)	$C_1(s) = K_P$ Kp = 3.2600
Integer Order PI controller (IOPI)	$C_1(s) = K_P + (K_I/s)$ Kp = 0.11525 $K_I = 0.0330$
Integer Order PD controller (IOPD)	$C_1(s) = K_P + sK_D$ Kp = 3.4512 $K_D = 0.0300$
Integer Order PID controller (IOPID)	$C_1(s) = K_P + (K_I/s) + sK_D$ Kp = 1.2647 $K_I = 0.4244$ $K_D = 0.0132$
Fractional Order PI controller (FOPI)	$C_1(s) = K_P + K_I/s^{\lambda}$ Kp = 1.0000 $K_I = 0.3000$ $\lambda = 0.9800$
Fractional Order PD controller (FOPD)	$C_1(s) = K_P + K_D s^{\mu}$ Kp = 18.1655 $K_D = 0.0994$ $\mu = 0.9537$
Fractional Order PID controller (FOPID)	$C_1(s) = K_P + K_I s^{-\lambda} + K_D s^{\mu}$ Kp = 2.4500 $K_I = 0.6000$ $K_D = 0.0580$ $\lambda = 0.9800$ $\mu = 0.9560$

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IV. Summary of the Simulation Results

The Performance of the various controllers are described in Table.3.

Here,

- C - Controller
- T_r - Rise Time
- T_s - Settling Time
- M_p - Overshoot
- V_m - Peak Amplitude
- Y/N– Checking the desired output condition.
- Y - Yes
- N – No

Table.3: Characteristics of Various Controllers

C	T_r(s)	T_s(s)	M_p(%)	V_m	Y/N
IOP	0.560	1.49	-	0.983	N
IOPI	10.70	32.80	5.38	1.050	Y
IOPD	0.542	1.44	-	0.986	N
IOPID	1.330	8.04	9.22	1.090	Y
FOPD	0.980	2.50	-	0.995	N
FOPI	2.900	10.00	4.40	1.044	Y
FOPID	1.910	6.00	1.90	1.019	Y

Thus the Plant with IOP, IOPD, FOPD Controllers don't give the desired output. And the plant with IOPI, IOPID, FOPI, FOPID Controllers give the desired output.

Table.4: Effect of Increasing a Parameter Independently

Parameter	Rise Time	Overshoot	Settling Time
K_P	Decreases	Increases	Small change
K_I	Decreases	Increases	Increases
K_D	Minor Decrease	Minor Decrease	Minor Decrease
λ	Decreases	Increases	Increases
μ	Decreases	Decreases	Decreases

Table.4 is obtained from the simulation results. K_I and λ have the same effect, likewise K_D and μ have the same effect.So by using small control input, it is easy to achieve desired results.

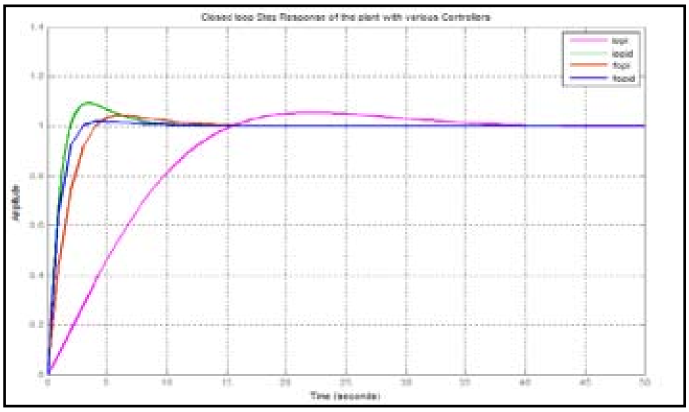

Fig.11: Closed loop step response of various controllers

V. Conclusion

Analyzing the stabilized inverted pendulum system under various controllers, the FOPID Controller is found to be the fastest response controller to reach stabilization. The single control affection of FOPID Controller may not be better than other controllers but its overall performance is much better than other controllers. The FOPID controller and IOPID can reach almost the same control affection. But comparing the overall performance it is proved that the FOPID controller is much better than other controllers. The future scope of this paper is to implement the FOPID controller in inverted pendulum system for better performance.

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