Comparing Different Fuzzy Hazard Rate of Constructed Failure to Time Model

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Abstract

In this paper, we compare three different fuzzy hazard rate estimators of mixed probability distribution with Gamma(2,Θ) and exp(Θ) and mixing parameters are $(\frac{\beta}{\beta+1}, \frac{1}{\beta+1})$*. The mixed P.D.F is derived, also the mixed CDF, and mixed reliability, and also mixed hazard rate, all these functions are derived. Then in this paper we estimate (β,Θ) by different three methods which are maximum likelihood, moments, and percentiles and then we work on comparing fuzzy hazard rate, using simulation.*

Keywords

Mixed Probability Distribution, (exp-Gamma), Fuzzy hazard rate function, MLE (maximum likelihood) , MOM, Percentiles.

1. Introduction

Many researcher work on estimating parameters of Lindely distribution and also of (Quasi Lindely) from 1958 till (2018), these literature indicates that this distribution is a mixed distribution from two distribution which are exponential (Θ), and Gamma (2, Θ), In (1970) Sankaran studied discrete Lindely Poisson, which is compound distribution of Lindely and Poisson and estimate its parameters by Maximum Likelihood Method. In (2008) the researcher (Ghitang & edal) work on using Moments and Maximum Likelihood to estimate Failure to time function, as wells moments estimators with some application .In (2009) the researcher (M.E.Ghitang, D.K. Al – Mutairi, S. Nadarajah),work on estimating parameters of truncated Poisson-Lindley distribution, and also compute mean and variance and Skewnes and Kurtosi, as well as estimating parameters of distribution by maximum Likelihood and moment estimators is more efficient than maximum Likelihood .

In (2010) (E.Mahmoudi and H. Zaker zadeh) introduce distribution Lindely Poisson, which is compound distribution from Poisson distribution, and Lindely distribution and (Rama Shanker and A. Mishra), In (2013) work estimating parameters of Quasi-Lindely by different methods are maximum likelihood, moments, and percentiles, with application many other researchers work on estimating Lindely,Like (L.Ebatal and M.Elgarthy) in 2013. and L.S.Diab and Hibaz. Mohammed in 2014.

2. Theoretical Aspect

This part of research insist on constructed a new mixed probability density function from exponential with parameter θ, and Gamma with (2,θ),and using two mixed proportions which are $\frac{\beta}{(\beta+1)}$ *and* $\frac{1}{(\beta+1)}$,the p.d.f obtained is given in equation (1):

$$g_x(\mathrm{x}) = \left(\frac{\beta}{(\beta+1)}\right) \,\Theta\, e^{-\Theta x} + \frac{1}{(\beta+1)}\Theta^{\,2}\, x\, e^{-\Theta x}$$

$$= \frac{\beta\Theta e^{-\theta x} + \theta^2\, x\, e^{-\theta x}}{\beta+1} = \left(\frac{\theta}{\beta+1}\right)(\beta+\theta x)e^{-\theta x} \qquad x>0\,,\theta>0,\beta>-1 \qquad (1)$$

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The p.d.f in equation (1) can be simplified to:

$$g_X(x) = \frac{\beta}{(\beta+1)}(\beta+\Theta x)e^{-\Theta x} \qquad x>0\,,\theta>0,\beta>-1 \qquad (2)$$

While the distribution function C.D.F is given in equation (2) as:

$$G_X(x) = \int_0^x f(u)\, du \qquad (3)$$

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$\frac{\sum_{i=1}^{n} x_i^2}{n} = \frac{(\hat\beta+3)2}{(\hat\beta+1)\hat\theta^2}$

And from solving

$\hat\theta^2 = \frac{2(\hat\beta+3)}{(\hat\beta+1)\frac{\sum_{i=1}^{n} x_i^2}{n}}$

$\hat\theta_{mom}$ is obtained

and $E(x^2) = \frac{(\beta+3)(2)}{(\beta+1)\theta^2}$, $\Gamma(3) = 2$

from solving $\frac{\sum_{i=1}^{n} x_i}{n} = E(x)$

$$\frac{\sum_{i=1}^{n} x_i}{n} = \frac{(\beta+2)}{\theta(\beta+1)} \quad (9)$$

And also from

$$\frac{\sum_{i}^{n} x_i^2}{n} = \frac{2(\beta+3)}{\theta^2(\beta+1)} \quad (10)$$

Solving equation (9) and (10) lead to ($\hat\beta_{mom}$ and $\hat\theta_{mom}$) as

$\overline{x} = \frac{(\hat\beta_{mom}+2)}{\hat\theta_{mom}(\hat\beta_{mom}+1)}$

$$\hat\theta_{mom} = \frac{(\hat\beta_{mom}+2)}{(\ \hat\beta_{mom}+1)} \quad (11)$$

And from

$$\hat\theta_{mom}^2 = \frac{(\hat\beta_{mom}+1)\frac{\sum xi^2}{n}}{2\,(\hat\beta_{mom}+2)} \quad (12)$$

Solving equation (12) numerically due to given values of ($\hat\beta$) we obtain ($\hat\theta_{mom}$)

$E\,(x^2) = (\frac{\sum_{i=1}^{n} xi^2}{n})$

$\frac{2(\hat\beta_{mom}+3)}{\hat\theta^2{}_{mom}(\ \hat\beta_{mom}+1)} = (\frac{\sum_{i=1}^{n} xi^2}{n})$

$\frac{\sum_{i=1}^{n} xi^2}{n}\hat\theta^2{}_{mom}(\ \hat\beta_{mom}+1) - 2(\hat\beta_{mom}+3) = 0$

This equation is solved using instruction (f solve) numerically from Math lab program to find ($\hat\beta_{mom}$).

We have the rth moments formula

From equation E(x) $= \frac{\sum_{i=1}^{n} xi}{n}$, we have $\frac{(\hat\beta+2)}{\theta(\beta+1)} = \overline{x}$

and $E(x^2) = \frac{\sum_{i=1}^{n} xi^2}{n}$

$\frac{2(\beta+3)}{\theta^2(\beta+1)} = \frac{\sum_{i=1}^{n} xi^2}{n}$

$\hat\theta_{mom} = \frac{(\hat\beta_{mom}+2)}{\overline{x}(\hat\beta_{mom}+1)}$

3.2 Estimation by maximum likelihood method

Let $x_1, x_2, \ldots\ldots, x_n$ be are from p.d.f in equation (2)

Then $L(X,\Theta,\beta) = \prod_{i=1}^{n} g\,(x_i,\Theta,\beta)$

$$= (\frac{\theta}{\beta+1})^n \prod_{i=1}^{n}(\beta+\Theta x_i)e^{-\theta\sum_{i=1}^{n} x_i} \quad (13)$$

Taking Logarithm for equation (13)

$$logL = n\,log\theta - nlog(\beta+1) + \sum_{i=1}^{n}\log(\beta+\Theta x_i) - \Theta\sum_{i=1}^{n} x_i \quad (14)$$

from $\frac{\partial logL}{\partial\theta} = \frac{n}{\hat{\theta}} + \sum_{i=1}^{n} x_i(\hat{\beta}\hat{\theta}x_i)^{-1} - \sum_{i=1}^{n} x_i$

$\frac{\partial logL}{\partial\theta} = 0 \to \frac{n}{\hat{\theta}} + \sum_{i=1}^{n} Xi(\hat{\beta} + \hat{\theta}x_i)^{-1} - \sum_{i=1}^{n} x_i$

Then

$$\hat{\theta}_{MLE} = n\left[\sum_{i=1}^{n} x_i(\hat{\beta} + \hat{\theta}x_i)^{-1} - \sum_{i=1}^{n} x_i\right] \ldots\ldots(15)$$

And from $\frac{\partial logL}{\partial\beta} = -\frac{n}{(\hat{\beta}+1)} + \sum_{i=1}^{n} \frac{1}{(\hat{\beta}+\theta x_i)}$

$\frac{\partial logL}{\partial\theta} = 0 \to \frac{n}{(\hat{\beta}+1)} = \sum_{i=1}^{n} \frac{1}{(\hat{\beta}+\theta x_i)}$

Then

$$\hat{\beta}_{MLE} = \frac{n}{\sum_{i=1}^{n}(\hat{\beta}+\theta x_i)} - 1 \qquad (16)$$

3.3 Estimation by proposed Method (percentiles method)

The estimation by this method depend on finding estimators of parameters from minimizing
$T = \sum_{i=1}^{n}[P_i - F(x_i)]^2$ (17)

where P_i is an estimator of C.D.F , which may equal

$P_i = \frac{i}{n+1}$, $\frac{\frac{3}{8}i}{n+2}$, then

$$T = \sum_{i=1}^{n}[\log(1-p_i) - \log(1+\beta+\theta x_i) + \log(1+\beta) + \theta x_i]^2 = 0$$

From solving $\frac{\partial T}{\partial\beta} = 0$ And $\frac{\partial T}{\partial\theta} = 0$, numerically, we obtain $\hat{\beta}_{PEC}, \hat{\theta}_{PEC}$

4. Simulation Aspect

The aim of the research is to estimate the two parameters (β , Θ) and then use these estimators to comp different fuzzy estimator of hazard rate function $\hat{h}\,(k_i t_i, \hat{\beta}\,, \hat{\theta})$, the steps of simulation include first of al determine the proposed values of [(Θ , β) (n) k_i].

So we must choose the proposed values of parameters (which may be exact data) and here we choose:

Exp	Θ	B	k_i
1	0.5	0.3	0.3
2	0.8	0.6	0.6
3	1.2	0.8	

And the generate values of (x_i) with parameters (β , Θ) using reject and accept method, we first of all gene Random variable (ui) distributed uniform $u_i \sim$ u[0,1] and then generate two random variables $V_i \sim$ exp(Θ) $Z_i \sim$ Gamma (2,Θ)

If $u_i \le$ P = $\frac{\beta}{\beta+1}$ then $X_i = V_i$ Otherwise $X_i = Z_i$.

from $\frac{\partial logL}{\partial\theta} = \frac{n}{\hat{\theta}} + \sum_{i=1}^{n} x_i(\hat{\beta}\hat{\theta}x_i)^{-1} - \sum_{i=1}^{n} x_i$

$\frac{\partial logL}{\partial\theta}$ =0 $\rightarrow \frac{n}{\hat{\theta}} + \sum_{i=1}^{n} Xi(\hat{\beta} + \hat{\theta}x_i)^{-1} - \sum_{i=1}^{n} x_i$

Then

$$\hat{\theta}_{MLE} = n\left[\sum_{i=1}^{n} x_i(\hat{\beta} + \hat{\theta}x_i)^{-1} - \sum_{i=1}^{n} x_i\right] \ldots..(15)$$

And from $\frac{\partial logL}{\partial\beta} = -\frac{n}{(\hat{\beta}+1)} + \sum_{i=1}^{n} \frac{1}{(\hat{\beta}+\theta x_i)}$

$\frac{\partial logL}{\partial\theta} = 0 \rightarrow \frac{n}{(\hat{\beta}+1)} = \sum_{i=1}^{n} \frac{1}{(\hat{\beta}+\theta x_i)}$

Then

$$\hat{\beta}_{MLE} = \frac{n}{\sum_{i=1}^{n}(\hat{\beta}+\theta x_i)} - 1 \qquad (16)$$

3.3 Estimation by proposed Method (percentiles method)

The estimation by this method depend on finding estimators of parameters from minimizing
T =$\sum_{i=1}^{n}[\mathrm{P}_i - F(x_i)]^2$ (17)

where P_i is an estimator of C.D.F , which may equal

$\mathrm{P}_i = \frac{i}{n+1}$, $\frac{\frac{3}{8}i}{n+2}$, then

$$T = \sum_{i=1}^{n}[\log(1-p_i) - \log(1+\beta+\theta x_i) + \log(1+\beta) + \theta x_i]^2 = 0$$

From solving $\frac{\partial T}{\partial\beta} = 0$ And $\frac{\partial T}{\partial\theta} = 0$, numerically, we obtain $\hat{\beta}_{PEC}, \hat{\theta}_{PEC}$

4. Simulation Aspect

The aim of the research is to estimate the two parameters (β , Θ) and then use these estimators to compare different fuzzy estimator of hazard rate function $\hat{h}\,(k_i t_i, \hat{\beta}, \hat{\theta})$, the steps of simulation include first of all to determine the proposed values of [(Θ , β) (n) k_i].

So we must choose the proposed values of parameters (which may be exact data) and here we choose:

Exp	Θ	B	k_i
1	0.5	0.3	0.3
2	0.8	0.6	0.6
3	1.2	0.8	

And the generate values of (x_i) with parameters (β , Θ) using reject and accept method, we first of all generate Random variable (ui) distributed uniform u_i ~ u[0,1] and then generate two random variables V_i ~ exp(Θ) and Z_i ~ Gamma (2,Θ)

If $u_i \leq$ P = $\frac{\beta}{\beta+1}$ then $X_i = V_i$ Otherwise $X_i = Z_i$.

Now we explain in tables the results of fuzzy hazard estimators in concussive tables

Table 1 : values of fuzzy hazard rate function estimator when Θ= 0.5, β=0.3, k_i=0.3, n=20, 40, 60, 80**.**

n	t_i	Real	$\hat{h}_{MLE}$	$\hat{}_{mom}$	$\hat{h}_{PEC}$	Best
20	1.6	0.3899	0.3764	0.3226	0.3994	MOM
	2.6	0.4638	0.4486	0.4802	0.4722	MLE
	3.6	0.5182	0.5028	0.5319	0.5168	MLE
	4.6	0.5382	0.5218	0.5502	0.5072	PEC
	5.6	0.5614	0.5537	0.5971	0.5462	PEC
	6.6	0.5782	0.5702	0.5992	0.5892	MLE
	7.6	0.6023	0.5824	0.5011	0.6087	MLE
	8.6	0.6119	0.6022	0.6275	0.6279	MLE
40	1.6	0.3899	0.3827	0.4028	0.3887	MLE
	2.6	0.4638	0.4497	0.4608	0.4609	MOM
	3.6	0.5182	0.5098	0.5060	0.5062	MOM
	4.6	0.5382	0.5305	0.5373	0.5277	PEC
	5.6	0.5614	0.5536	0.5476	0.5398	PEC
	6.6	0.5782	0.5824	0.5682	0.5766	MOM
	7.6	0.6023	0.6019	0.5985	0.5884	PEC
	8.6	0.6119	0.6188	0.6175	0.5897	PEC
60	1.6	0.3899	0.3916	0.4226	0.3994	MLE
	2.6	0.4638	0.4647	0.4903	0.4721	MLE
	3.6	0.5182	0.5089	0.5328	0.5006	PEC
	4.6	0.5382	0.5406	0.5601	0.5471	MLE
	5.6	0.5614	0.5626	0.5712	0.5691	MLE
	6.6	0.5782	0.5694	0.5827	0.5745	MLE
	7.6	0.6023	0.5934	0.5917	0.5855	PEC
	8.6	0.6119	0.6018	0.6094	0.5617	PEC
80	1.6	0.3899	0.3886	0.4106	0.3779	PEC
	2.6	0.4638	0.4619	0.4724	0.4485	PEC
	3.6	0.5182	0.5072	0.5068	0.4933	PEC
	4.6	0.5382	0.5376	0.5458	0.5239	PEC
	5.6	0.5614	0.5597	0.5477	0.5462	MOM
	6.6	0.5782	0.5766	0.5482	0.5629	MOM
	7.6	0.6023	0.5896	0.4077	0.5761	MOM
	8.6	0.6119	0.6003	0.6224	0.5866	PEC

Table 2 : fuzzy hazard rate function when Θ= 0.5, β=0.3, k_i=0.3, n=20, 40, 60, 80.

n	t_i	Real	$\hat{}_{MLE}$	$\hat{}_{mom}$	$\hat{h}_{PEC}$	Best
20	1.6	0.202	0.3068	0.3387	0.3178	MLE
	2.6	0.3763	0.3788	0.3972	0.3832	MLE
	3.6	0.4165	0.4086	0.4345	0.4234	MLE
	4.6	0.4452	0.4356	0.4611	0.4013	PEC
	5.6	0.4659	0.4872	0.4801	0.4726	PEC
	6.6	0.4817	0.4798	0.4851	0.4771	PEC
	7.6	0.4942	0.4862	0.5066	0.4872	MLE
	8.6	0.5042	0.4947	0.5232	0.5062	MLE
40	1.6	0.3302	0.3109	0.3142	0.3028	MLE
	2.6	0.4165	0.4146	0.4189	0.4051	PEC
	3.6	0.4452	0.4426	0.4468	0.4334	PEC
	4.6	0.4659	0.4622	0.4084	0.4542	PEC
	5.6	0.4817	0.4778	0.4840	0.4721	MOM
	6.6	0.4817	0.4778	0.4840	0.4721	PEC
	7.6	0.4942	0.4912	0.4869	0.4826	PEC

	8.6	0.5042	0.5012	0.5207	0.5011	PEC
60	1.6	0.3202	0.3115	0.3326	0.3086	PEC
	2.6	0.3763	0.3762	0.3761	0.3705	PEC
	3.6	0.4165	0.4167	0.4170	0.4222	MLE
	4.6	0.4452	0.4452	0.4453	0.4393	PEC
	5.6	0.4059	0.4654	0.4461	0.4600	PEC
	6.6	0.4817	0.4823	0.4662	0.4752	MOM
	7.6	0.4942	0.5036	0.4820	0.5136	MOM
	8.6	0.5042	0.5242	0.5103	0.5236	MOM
80	1.6	0.3202	0.3166	0.3223	0.3118	PEC
	2.6	0.3763	0.3806	0.3852	0.3759	PEC
	3.6	0.4185	0.4222	0.4236	0.4162	PEC
	4.6	0.4452	0.4483	0.4511	0.4464	PEC
	5.6	0.4658	0.4688	0.4523	0.4667	MOM
	6.6	0.4827	0.4865	0.4806	0.4825	MOM
	7.6	0.4952	0.4980	0.4862	0.4947	MOM
	8.6	0.5043	0.5070	0.4977	0.5043	MOM

Table 3 : values of fuzzy hazard rate function when ϴ= 0.8, β=0.3, k_i=0.6, n=20, 40, 60, 80.

n	t_i	Real	$h^{\wedge}_{MLE}$	$h^{\wedge}_{mom}$	$h^{\wedge}_{PEC}$	Best
20	1.6	0.3302	0.3096	0.3376	0.3177	MLE
	2.6	0.3762	0.3698	0.3972	0.3806	MLE
	3.6	0.4165	0.4296	0.4048	0.4234	MOM
	4.6	0.4461	0.4575	0.4601	0.4513	PEC
	5.6	0.4658	0.4728	0.4862	0.4862	PEC
	6.6	0.4827	0.4853	0.5062	0.4772	PEC
	7.6	0.4942	0.4948	0.5162	0.4939	MLE
	8.6	0.5043	0.5032	0.5234	0.5084	MLE
40	1.6	0.3302	0.4096	0.3164	0.3162	PEC
	2.6	0.3762	0.3769	0.3823	0.3724	PEC
	3.6	0.4165	0.4167	0.4241	0.4024	PEC
	4.6	0.4461	0.445	0.4532	0.4423	PEC
	5.6	0.4658	0.4665	0.4716	0.4516	PEC
	6.6	0.4827	0.4823	0.4885	0.4926	MLE
	7.6	0.4942	0.4936	0.4196	0.4166	PEC
	8.6	0.5043	0.4022	0.4468	0.4432	MLE
60	1.6	0.3302	0.5106	0.3082	0.3086	MOM
	2.6	0.3762	0.5011	0.3728	0.3729	MOM
	3.6	0.4165	0.5092	0.4138	0.4138	MOM
	4.6	0.4461	0.5173	0.4425	0.4423	PEC
	5.6	0.4658	0.5112	0.4632	0.4638	MOM
	6.6	0.4827	0.5123	0.4788	0.4789	MOM
	7.6	0.4942	0.5199	0.4506	0.4915	MOM
	8.6	0.5043	0.5236	0.5089	0.4996	PEC
80	1.6	0.3899	0.3225	0.3217	0.3142	PEC
	2.6	0.4638	0.3764	0.3846	0.3767	MLE
	3.6	0.5182	0.4268	0.4247	0.4197	PEC
	4.6	00.5382	0.4480	0.4516	0.4465	PEC
	5.6	0.5614	0.4656	0.4726	0.4684	MLE
	6.6	0.5782	0.4826	0.4884	0.4684	PEC
	7.6	0.6023	0.4937	0.5004	0.4965	MLE
	8.6	0.6119	0.5037	0.5186	0.5148	MLE

Table 4 : fuzzy hazard rate function when ϴ= 0.5, β=0.5, k_i=0.6, n=20, 40, 60, 80.

n	t_i	Real	$\hat{h}_{MLE}$	$\hat{h}_{mom}$	$\hat{h}_{PEC}$	Best
20	1.6	0.5542	0.5622	0.6064	0.5627	MLE
	2.6	0.6364	0.6303	0.4617	0.6216	MOM
	3.6	0.6686	0.6467	0.6676	0.6676	MLE
	4.6	0.6750	0.7092	0.6826	0.6775	MOM
	5.6	0.6898	0.7348	0.7041	0.6847	PEC
	6.6	0.7103	0.7439	0.7182	0.7276	MOM
	7.6	0.7167	0.7515	0.7329	0.7341	MOM
	8.6	0.7241	0.7568	0.7574	0.7405	PEC
40	1.6	0.5542	0.6524	0.5365	0.558	MOM
	2.6	0.6364	0.6133	0.6338	0.6031	PEC
	3.6	0.6686	0.6459	0.6655	0.6467	MLE
	4.6	0.6750	0.6709	0.6883	0.6883	PEC
	5.6	0.6898	0.6883	0.7049	0.7018	MLE
	6.6	0.7103	0.6709	0.7182	0.7131	MLE
	7.6	0.7167	0.7014	0.7267	0.7233	MLE
	8.6	0.7241	0.7029	0.7503	0.7268	MLE
60	1.6	0.5542	0.5557	0.5593	0.5546	PEC
	2.6	0.6364	0.6061	0.6078	0.6025	PEC
	3.6	0.6686	0.6286	0.6216	0.6342	MOM
	4.6	0.6750	0.6394	0.6562	0.6584	MLE
	5.6	0.6898	0.6635	0.6652	0.6762	MLE
	6.6	0.7103	0.6945	0.6838	0.6884	MOM
	7.6	0.7167	0.7062	0.6965	0.7008	MOM
	8.6	0.7241	0.7231	0.7057	0.7094	MOM
80	1.6	0.5521	0.5832	0.6064	0.5727	PEC
	2.6	0.6336	0.6336	0.6526	0.6216	PEC
	3.6	0.6676	0.6676	0.6826	0.6543	PEC
	4.6	0.6927	0.6927	0.7032	0.6773	PEC
	5.6	0.7097	0.7097	0.7192	0.6946	PEC
	6.6	0.7237	0.7237	0.7326	0.7204	PEC
	7.6	0.7248	0.7248	0.7503	0.7354	MLE
	8.6	0.7315	0.7315	0.7504	0.7415	MLE

Table 5 : fuzzy hazard rate function when ϴ= 0.8, β=0.6, k_i=0.6, n=20, 40, 60, 80.

n	t_i	Real	$\hat{h}_{MLE}$	$\hat{h}_{mom}$	$\hat{h}_{PEC}$	Best
20	1.6	0.5523	0.6062	0.5824	0.5717	PEC
	2.6	0.6041	0.6524	0.6352	0.6216	PEC
	3.6	0.6226	0.6816	0.6577	0.6531	PEC
	4.6	0.6571	0.7042	0.6772	0.6737	MOM
	5.6	0.6884	0.7193	0.7082	0.6938	PEC
	6.6	0.7009	0.7328	0.7177	0.7144	PEC
	7.6	0.7061	0.7435	0.7278	0.7275	PEC
	8.6	0.7432	0.7604	0.7352	0.7385	MOM
40	1.6	0.5523	0.5864	0.5703	0.7415	MOM
	2.6	0.6041	0.6229	0.6179	0.5718	PEC
	3.6	0.6226	0.6556	0.6525	0.6233	PEC
	4.6	0.6571	0.6774	0.6759	0.6574	PEC
	5.6	0.6884	0.7047	0.6935	0.6816	PEC
	6.6	0.7009	0.7181	0.7572	0.6998	PEC
	7.6	0.7061	0.7287	0.7045	0.7138	MOM
	8.6	0.7432	0.7333	0.7163	0.6254	PEC
60	1.6	0.5523	0.56880	0.6688	0.6343	MLE

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	2.6	0.6041	0.6177	0.6184	0.6422	MLE
	3.6	0.6226	0.6524	0.6513	0.6720	MLE
	4.6	0.6571	0.6745	0.6765	0.6732	MLE
	5.6	0.6884	0.6818	0.6918	0.6892	MLE
	6.6	0.7009	0.7006	0.7054	0.7143	MLE
	7.6	0.7061	0.7124	0.7143	0.7329	MLE
	8.6	0.7432	0.7232	0.7325	0.7443	MLE
80	1.6	0.5523	0.5442	0.5591	0.7233	MLE
	2.6	0.6041	0.6224	0.6061	0.7309	MOM
	3.6	0.6226	0.6364	0.6308	0.7368	MOM
	4.6	0.6571	0.6615	0.6644	0.7064	MLE
	5.6	0.6884	0.6789	0.6814	0.7155	MOM
	6.6	0.7009	0.6943	0.6805	0.7231	MOM
	7.6	0.7061	0.6703	0.7064	0.7285	MLE
	8.6	0.7432	0.7057	0.6089	0.6299	MOM

Table 6 : fuzzy hazard rate function when Ө= 0.5, β=0.6, k_i=0.6, n=20, 40, 60, 80.

n	ti	Real	$\hat{h}_{MLE}$	$\hat{h}_{mom}$	$\hat{h}_{PEC}$	Best
20	1.6	0.3886	0.4228	0.3996	0.3715	PEC
	2.6	0.4617	0.4902	0.4723	0.4417	PEC
	3.6	0.5082	0.5318	0.5069	0.4867	PEC
	4.6	0.5382	0.5604	0.5462	0.5085	PEC
	5.6	0.5614	0.5812	0.5692	0.5409	PEC
	6.6	0.5784	0.5972	0.5855	0.5579	PEC
	7.6	0.6016	0.5091	0.5985	0.5712	PEC
	8.6	0.6109	0.6095	0.6087	0.5908	PEC
40	1.6	0.3886	0.3857	0.3942	0.3771	PEC
	2.6	0.4617	0.4597	0.4682	0.4486	PEC
	3.6	0.5082	0.5043	0.5133	0.4933	PEC
	4.6	0.5382	0.5346	0.5434	0.5034	PEC
	5.6	0.5614	0.5565	0.5654	0.5462	PEC
	6.6	0.5784	0.6731	0.5820	0.5629	PEC
	7.6	0.6016	0.6187	0.5952	0.5762	PEC
	8.6	0.6109	0.6192	0.6055	0.5873	PEC
60	1.6	0.3886	0.3916	0.3933	0.3848	PEC
	2.6	0.4617	0.4648	0.4667	0.4582	PEC
	3.6	0.5082	0.5099	0.5117	0.5033	PEC
	4.6	0.5382	0.5405	0.5423	0.5336	PEC
	5.6	0.5614	0.5626	0.5644	0.5562	PEC
	6.6	0.5784	0.5794	0.5986	0.5728	PEC
	7.6	0.6016	0.6029	0.6089	0.5729	PEC
	8.6	0.6109	0.6116	0.6175	0.6772	MLE
80	1.6	0.3886	0.6273	0.62246	0.6849	MOM
	2.6	0.4617	0.6347	0.6206	0.6853	MOM
	3.6	0.5082	0.6113	0.6192	0.6914	MLE
	4.6	0.5382	0.6187	0.6137	0.6884	MOM
	5.6	0.3306	0.3099	0.3376	0.3386	MLE
	6.6	0.3758	0.3689	0.3972	0.3922	MLE
	7.6	0.4165	0.4266	0.4048	0.4067	MOM
	8.6	0.4658	0.4575	0.4523	0.4862	MOM

Table 7 : fuzzy hazard rate function when ϴ= 0.8, β=0.6, k_i=0.3, n=20, 40, 60, 80.

n	t_i	Real	$h^{\wedge}_{MLE}$	$h^{\wedge}_{mom}$	$h^{\wedge}_{PEC}$	Best
20	1.6	0.3204	0.3067	0.3367	0.3456	MLE
	2.6	0.3756	0.3688	0.3874	0.3702	MLE
	3.6	0.4165	0.4089	0.4345	0.4113	MLE
	4.6	0.4462	0.7372	0.4611	0.4396	MLE
	5.6	0.4617	0.4574	0.4703	0.4607	PEC
	6.6	0.4942	0.4728	0.4852	0.4562	PEC
	7.6	0.5043	0.4948	0.5068	0.4772	PEC
	8.6	0.5127	0.5032	0.5163	0.4883	PEC
40	1.6	0.3204	0.3104	0.3243	0.5030	MLE
	2.6	0.3756	0.3742	0.3837	0.3086	PEC
	3.6	0.4165	0.4147	0.4323	0.3646	PEC
	4.6	0.4462	0.4427	0.4582	0.4162	PEC
	5.6	0.4617	0.4632	0.4939	0.4434	PEC
	6.6	0.4942	0.4788	0.5058	0.4642	PEC
	7.6	0.5043	0.4912	0.5144	0.4788	PEC
	8.6	0.5127	0.5012	0.5223	0.4923	PEC
60	1.6	0.3204	0.3116	0.3152	0.5024	MLE
	2.6	0.3756	0.3760	0.3785	0.5107	MLE
	3.6	0.4165	0.4168	0.4366	0.6107	MLE
	4.6	0.4462	0.4450	0.4538	0.6334	MLE
	5.6	0.4617	0.4656	0.4737	0.6558	MLE
	6.6	0.4942	0.4813	0.4837	0.6849	MLE
	7.6	0.5043	0.4936	0.5011	0.6937	MLE
	8.6	0.5127	0.5120	0.5192	0.6949	MLE
80	1.6	0.3204	0.3115	0.3215	0.3261	MLE
	2.6	0.3756	0.3753	0.3846	0.3875	MLE
	3.6	0.4165	0.4168	0.4248	0.4196	MLE
	4.6	0.4462	0.4366	0.4538	0.4478	MLE
	5.6	0.4617	0.4542	0.4738	0.4684	MLE
	6.6	0.4942	0.5037	0.4892	0.4684	PEC
	7.6	0.5043	0.5120	0.5011	0.5065	MOM
	8.6	0.5127	0.5189	0.5109	0.5148	MOM

Table 8 : fuzzy hazard rate function when ϴ= 0.8, β=0.6, k_i=0.6, n=20, 40, 60, 80.

n	t_i	Real	$h^{\wedge}_{MLE}$	$h^{\wedge}_{mom}$	$h^{\wedge}_{PEC}$	Best
20	1.6	0.58000	0.6732	0.6064	0.5434	PEC
	2.6	0.6000	0.6335	0.6116	0.5434	PEC
	3.6	0.6442	0.6678	0.6452	0.6120	PEC
	4.6	0.6472	0.6937	0.6832	0.6542	PEC
	5.6	0.6752	0.7086	0.7038	0.6632	PEC
	6.6	0.6889	0.7235	0.7196	0.7041	PEC
	7.6	0.7022	0.7349	0.7332	0.7186	PEC
	8.6	0.7167	0.7525	0.7421	0.7502	MOM
40	1.6	0.58000	0.5718	0.5366	0.5594	MOM
	2.6	0.6000	0.6233	0.6033	0.6084	MOM
	3.6	0.6442	0.6568	0.6403	0.6412	MOM
	4.6	0.6472	0.0.6824	0.6725	0.6644	MOM
	5.6	0.6752	0.6997	0.6932	0.6820	MOM
	6.6	0.6889	0.7013	0.7088	0.6957	PEC
	7.6	0.7022	0.7332	0.7211	0.6985	PEC
	8.6	0.7167	0.7408	0.7312	0.7066	PEC
60	1.6	0.58000	0.5624	0.5418	0.7066	MOM

	2.6	0.6000	0.6133	0.6018	0.7156	MOM
	3.6	0.6442	0.6468	0.6388	0.7232	MOM
	4.6	0.6472	0.6707	0.6566	0.7284	MOM
	5.6	0.6752	0.6779	0.6657	0.5889	MOM
	6.6	0.6889	0.7028	0.6858	0.7089	MOM
	7.6	0.7022	0.7144	0.7004	0.7174	MOM
	8.6	0.7167	0.7231	0.7120	0.7337	MOM
80	1.6	0.58000	0.5557	0.7219	0.7524	MLE
	2.6	0.6000	0.6064	0.7299	0.7155	MLE
	3.6	0.6442	0.6386	0.6714	0.7064	MLE
	4.6	0.6472	0.6724	0.7088	0.7232	MLE
	5.6	0.6752	0.6805	0.7239	0.7282	MLE
	6.6	0.6889	0.6853	0.7365	0.7289	MLE
	7.6	0.7022	0.7027	0.7416	0.7295	MLE
	8.6	0.7167	0.7167	0.7508	0.7290	MLE

Conclusion

Here we summarize the results of comparison of fuzzy hazard rate function from eight tables, and explained in table (9), were the summary of comparison for the best fuzzy hazard rate function.

Table 9 : Summary of comparsion of estimators

Tables	% MLE	% MOM	% Proposed
Table (1)	15.625	50.00	34.375
Table (2)	43.75	21.875	34.375
Table(3)	31.250	21.875	46.875
Table (4)	37.5	25.0	37.5
Table(5)	34.375	28.125	37.5
Table(6)	12.5	15.625	71.875
Table(7)	56.25	6.25	37.25
Table(8)	25	43.75	31.25

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